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A TWO-WAREHOUSE INVENTORY MODEL WITH REWORK PROCESS AND TIME-VARYING DEMAND

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Abstract: A two-warehouse inventory model with deteriorating items and rework process with time-varying demand rate is presented. The last-in-first-out (LIFO) and first-in-first-out (FIFO) policies are considered with the assumption that the holding cost is higher in the rented warehouse (RW) compared to the owned warehouse (OW). The aim of the proposed model is to ascertain the optimum values of time in a production cycle that will minimise the total relevant cost, TRC*. We have utilised Microsoft Excel Solver as a solution tool, in which the generalised reduced gradient (GRG Nonlinear) method has been chosen as the solving method. The result is further verified using the built-in function in the Mathematica software. We observed that given same changes made to the parameters in both the LIFO and FIFO systems, a lower TRC* is obtained in the former. This shall mean that the LIFO system is less expensive than the FIFO system, provided that the holding cost in RW is higher than that in OW. The flow of inventory in the LIFO system suggests that items stored last in the OW will be dispatched first. This is an important factor for manufacturers for ensuring that items are distributed at optimal freshness.

Keywords: *Two-warehouse model, LIFO policy, FIFO policy, rework process, deterioration*

1. Introduction

In the classical inventory model, a single-warehouse system is generally considered where the capacity of the warehouse is known and limited. A single warehouse is more suitable for small businesses in which it is able to accommodate sufficient stock for their operation. However, for large businesses, a supplementary storage facility with a large space capacity is essential to hold excess inventories due to several factors such as temporary discounts and launching of new products. In order to manage the volatility in demand, companies often use rented warehouses (RWs) along with owned warehouses (OWs) to stock inventories sufficiently over time to absorb any fluctuations in demand (Kumar et al., 2018). Therefore, it is essential to focus on the inventory problem with more than just one warehouse, which will make the inventory system hold greater validity and relevance in real-life situations.

Over the years, numerous researchers have considered the two-warehouse system in their inventory models. The two-warehouse inventory model was first introduced by Hartley (1976) with the assumption that the holding cost in RW is greater than that in OW. Sarma (1983) proposed the inventory model with two-level storage and optimum release rule. Goswami et al. (1992) proposed an economic quantity model with two warehouses under time-varying linearly increasing demand. They assumed that transportation costs from RW to OW are proportional to the quantity transported and that items are delivered in an irregular pattern from RW to OW. Panda et al. (2010) also considered two-warehouse inventory models and focused on multiple retailers with price- and stock-dependent demand.

Another common and unrealistic assumption in the classical inventory model is that the received items are of perfect quality. However, in real-life situations, this assumption may not always be true. Realistically, items such as food, cosmetics, medicine, among others deteriorate over time. The existence of deterioration was first considered by Ghare et al. (1963). Rafaat (1991) also conducted a survey on the literature relating to inventory models with deteriorating items. Singh et al. (2013) had further incorporated an imperfect production process where the demand rate is assumed to be time dependent, while the production rate is dependent on the demand rate. Agrawal et al. (2013) considered the presence of deteriorating items in their model and provided the option to choose between a single- or two-warehouse system. They concluded that the cost acquired at OW due to high deterioration rate could be balanced out by purchasing more items to be stored in RW, hence reducing the shortage cost.

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Throughout the years, several other researchers such as Pakkala et al. (1991), Bankherouf (1997), Lee et al. (2000), Wee et al. (2005), Rong et al. (2008), Lee et al. (2009), Panda et al. (2012), Yadav et al. (2013), Bhunia et al. (2014), Kaliraman et al. (2017), Chakrabarty et al. (2018), Shaikh et al. (2019), Indrajitsingha et al. (2019), Aastha et al. (2020), and Gupta et al. (2020) have considered two-warehouse inventory models for deteriorating items with different types of demand.

It is generally assumed that the RW offers better preserving facilities than the OW; therefore, it charges a higher holding cost (Lee, 2006). The two-warehouse inventory models discussed above naturally adopt the LIFO (last-in-first-out) inventory policy. However, their studies do not focus or highlight the aforementioned point. The inventory policy needs to be further investigated, especially in the handling of deteriorating items.

Motivated from this significant point, Lee (2006) considered the two-warehouse inventory problem for deteriorating items and modified the LIFO model by Pakkala et al. (1992). Items stored later in RW will be utilised prior to those stored in OW in this model. Lee (2006) further proposed a model with a policy that is the opposite of the LIFO phenomenon, known as the FIFO (first-in-first-out) policy. In this model, items that are stored in OW first will be the first to be exhausted. It was concluded that the key in choosing between the two mentioned models are the deterioration rates and holding costs.

Similarly, Xu et al. (2017) considered a constant demand rate with deteriorating items over a finite time horizon in their two-warehouse inventory problem. They compared their model with the LIFO, modified LIFO, and FIFO inventory models and derived the critical conditions of the production cycle number, inventory holding and deterioration costs in the two warehouses.

Unlike other studies on LIFO and FIFO policies, Alamri et al. (2008) proposed a new policy named allocation-in-fraction-out (AIFO). The policy implies that inventory in both warehouses experience simultaneous consumption fractions, which indicate that the items are depleted by the end of the same cycle.

Introducing a rework process in an inventory model would allow the reduction of the costs involved in a production process. Wee et al. (2012) developed an economic production quantity model for deteriorating items with rework and stochastic preventive maintenance time. LIFO policy and lost sales were also considered in their study. Wee et al. (2013) then developed a production model using the FIFO rule for deteriorating items with stochastic preventive maintenance time and rework process. They assumed that the deterioration rates for both serviceable and recoverable items are the same.

Chung et al. (2009) suggested an inventory model that incorporates a two-warehouse system and the existence of defectives due to an imperfect quality production process. The defectives are assumed to be sold as a single batch at a discounted price. Yu (2009) developed an inventory model with deteriorating loss, shortage backordering, and trade credit with the aim to optimise the two-echelon system. A supplier and a distributor are considered in the two-warehouse environment system where the rental cost of the rented warehouse decreases over time. The study implied that coordination among distributors and suppliers is necessary to reduce total costs.

Ghiami et al. (2020) adopted the conventional logic of the OW and RW method into their study. They modelled a two-warehouse supply chain for a deteriorating product involving a retailer and a wholesaler. The retailer's main store or shelf is considered to be the OW, while the back-room for keeping extra stock is the RW. A continuous resupply FIFO policy is applied between the two-warehouses.

Considering the gaps within the study area where the aforementioned factors are considered simultaneously, the aim of this study is to develop a two-warehouse model by incorporating the LIFO and FIFO policies while considering a rework process. Our first approach is to consider an increasing demand rate instead of the commonly used constant demand rate. This approach would allow inventory operators to plan their production accordingly when launching new items into the market. Following the current trend, we could see that a newly launched product will experience a linearly increasing demand rate at the beginning of the launching period to a certain extent. A different approach on the purpose of RW has been incorporated in this paper, where the space is utilised to store finished products from a rework process with special requirements.

The second dilemma we have encountered is that some of the existing studies done have only considered a perfect production process. In other words, the presence of defective items is overlooked. Therefore, we have included a more realistic condition by introducing an imperfect production process, hence producing defective items. Aiming to lower the total relevant cost of the inventory model, a rework process is introduced in this study. In addition, we have proposed to separate perfect items from the defectives and assumed that the items are being repaired or will undergo the rework process only in the RW, once the production period has ended. This would be convenient and beneficial to manufacturers who have limited resources such as machines or operators, as they are

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able to focus on the production process first and the rework process later.

The final motivating factor would be the commonly used assumption that a storage facility or warehouse has an unlimited capacity. This is unrealistic as a storage space would be quickly filled up during an ongoing production process. Hence, we have included an approach where the OW would have limited capacity. Once the OW has reached its maximum capacity, excess items may be stored in a second warehouse, known as the rented warehouse.

In summary, we have proposed a two-warehouse inventory model with deteriorating items and a rework process for time-varying demand rate problem. The LIFO and FIFO policies are also incorporated in our model.

2. Mathematical Formulations

2.1 Notations

Listed below are the notations used in the models discussed in this paper.

$f(t)$	linearly increasing demand rate $f(t) = a + bt$, where a is the initial inventory level and b is the gradient for the demand function
P	constant production rate, units per unit time where $P > f(t)$ for all t
R	rework process rate, units per unit time where $R > f(t)$ for all t
x	product defect rate, units per unit time
α	deteriorating rate in OW and RW, units per unit time where $0 \leq \alpha \leq 1$
S	production setup cost, \$ per setup
c_P	production processing cost per unit item, \$
c_R	rework processing cost per unit item, \$
c_D	deterioration cost per unit item, \$
t_i	time for each stage
T	batch cycle time period
$I_i(t)$	inventory level at time t_i where $0 \leq i \leq 5$ for LIFO policy and $0 \leq i \leq 6$ for FIFO policy
h_1, h_2	holding costs per unit item, \$ in OW and RW, respectively
W	maximum inventory of OW, items
Q	maximum inventory of RW, items
TRC	total relevant costs per unit time, \$

2.2 Assumptions

The following are the assumptions adopted in this study:

(i) Lead time is zero, while the replenishment rate is finite.
(ii) RW has an unlimited capacity.
(iii) The perfect items from the production process are stored in OW, while the imperfect items from the rework process are sent to RW. The inventories in both warehouses decrease due to deterioration of items and fulfilment of demand.
(iv) The rework process is assumed to be perfect since special care is given to the process. Hence, all items that have been reworked are assumed to be perfect. The total number of reworked items is equal to the total defective items from the normal production process.
(v) The production rate, P, and rework rate, R, are assumed to be different.

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2.3 LIFO Policy

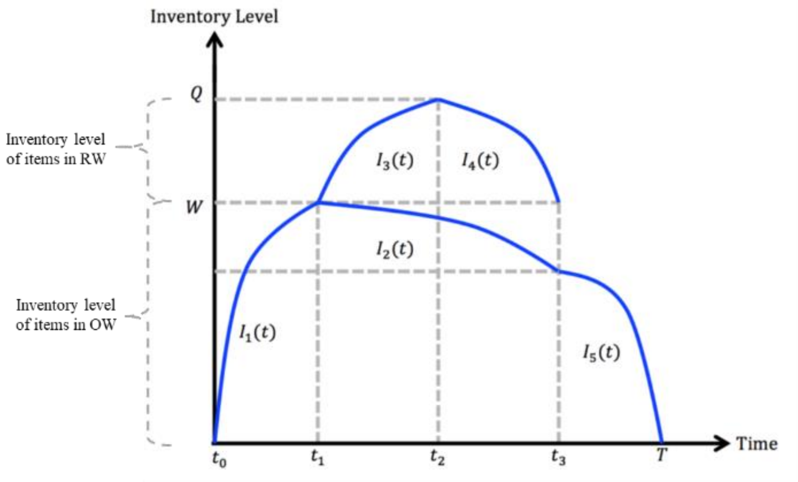

Figure 1. Inventory level with LIFO policy

Figure 1 illustrates the production system of a two-warehouse model with LIFO policy. The inventory stage can be divided into four intervals separated by $\boldsymbol{t_1}$, $\boldsymbol{t_2}$, $\boldsymbol{t_3}$, and $\boldsymbol{T}$.

The production cycle begins at $\boldsymbol{t_0}$, where fulfilment of demand and deterioration occurs simultaneously in the first interval. Since the production process is assumed to be imperfect, defective items are produced in this interval. The defectives are separated and kept aside to be sent to RW at $\boldsymbol{t_1}$. The differential equation representing the change of inventory level in OW during the interval $\boldsymbol{t_0 \leq t \leq t_1}$ is represented by:

$$\frac{dI_1(t)}{dt} = P - x - f(t) - \alpha I_1(t) \qquad ;\ t_0 \leq t \leq t_1 \qquad (1)$$

By solving differential equation (1) with the initial condition, $\boldsymbol{I_1(t_0) = 0}$, the inventory level in this interval is given as:

$$I_1(t) = \left[\frac{(P - x - a)}{\alpha} + \frac{b}{\alpha^2}\right](1 - e^{-\alpha t}) - \frac{bt}{\alpha} \qquad (2)$$

While defective items were sent to RW to undergo rework process, items in OW are kept idle or on standby during the interval $\boldsymbol{t_1 \leq t \leq t_3}$. However, due to deterioration, items will deplete at the rate $\boldsymbol{\alpha}$ in which the change of inventory level is represented by:

$$\frac{dI_2(t)}{dt} = -\alpha I_2(\mathrm{t}) \qquad ;\ t_1 \leq t \leq t_3 \qquad (3)$$

Considering the boundary condition $\boldsymbol{I_2(t_1) = W}$, where $\boldsymbol{W}$ represents the maximum capacity of OW, we have the following inventory level:

$$I_2(t) = We^{\alpha(t_1 - t)} \qquad (4)$$

The rework process in RW begins in the interval $\boldsymbol{t_1 \leq t \leq t_2}$, where items are depleted due to deterioration and fulfilment of demand. Hence, the differential equation representing the change of inventory level during this interval is denoted by:

$$\frac{dI_3(t)}{dt} = R - f(t) - \alpha I_3(\mathrm{t}) \qquad ;\ t_1 \leq t \leq t_2 \qquad (5)$$

Considering the boundary condition $\boldsymbol{I_3(t_1) = 0}$, the inventory level in this interval is denoted by:

$$I_3(t) = \frac{1}{\alpha}(R - a - bt) + \frac{b}{\alpha^2} - \left[\frac{1}{\alpha}(R - a - bt_1) + \frac{b}{\alpha^2}\right]e^{\alpha(t_1 - t)} \qquad (6)$$

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The rework process in RW ends at $\boldsymbol{t_2}$, in which the reworked items are assumed to be as good as new. The items will now deplete in the interval $\boldsymbol{t_2 \le t \le t_3}$ due to deterioration and fulfilment of demand and eventually utilised completely at $\boldsymbol{t_3}$. The change of inventory level is represented by:

$$\frac{dI_4(t)}{dt} = -f(t) - \alpha I_4(\mathrm{t}) \qquad ;\ t_2 \le t \le t_3 \qquad (7)$$

Considering the boundary condition $\boldsymbol{I_4(t_3) = 0}$, the inventory level in this interval is denoted by:

$$I_4(t) = \left[\frac{1}{\alpha}(a + bt_3) - \frac{b}{\alpha^2}\right] e^{\alpha(t_3 - t)} - \frac{(a + bt)}{\alpha} + \frac{b}{\alpha^2} \qquad (8)$$

Once the items in RW are completely depleted at $\boldsymbol{t_3}$, fulfilment of demand and deterioration will be covered by the remaining inventory in OW during the last interval $\boldsymbol{t_3 \le t \le T}$. The production cycle ends at $\boldsymbol{T}$, in which the inventory level in OW reaches zero and is completely utilised. The change in inventory level in this interval is denoted by:

$$\frac{dI_5(t)}{dt} = -f(t) - \alpha I_5(\mathrm{t}) \qquad ;\ t_3 \le t \le T \qquad (9)$$

Considering the boundary condition $\boldsymbol{I_5(T) = 0}$, the inventory level in this interval is denoted by:

$$I_5(t) = \left[\frac{(a + bT)}{\alpha} - \frac{b}{\alpha^2}\right] e^{\alpha(T - t)} - \frac{(a + bt)}{\alpha} + \frac{b}{\alpha^2} \qquad (10)$$

Note that items in RW that entered the production cycle last are the items to be utilised first, hence the name ‘last-in-first-out’ policy.

The maximum inventory of OW, $\boldsymbol{W}$, is governed by equation $\boldsymbol{W = I_1(t_1)}$; hence, we have the following equation:

$$W = \left[\frac{(P - x - a)}{\alpha} + \frac{b}{\alpha^2}\right](1 - e^{-\alpha t_1}) - \frac{bt_1}{\alpha} \qquad (11)$$

Next, let $\boldsymbol{A_i}$ denote the time-weighted inventory for each interval, where $\boldsymbol{i = 1, 2, \ldots, 5}$, and it represents the amount of inventory under the curve $\boldsymbol{I_i(t)}$. The following are the respective equations for $\boldsymbol{A_i}$ in the corresponding intervals:

In the interval $\boldsymbol{t_0 \le t \le t_1}$,

$$\begin{aligned} A_1 &= \int_0^{t_1} I_1(t)\, dt \\ &= \frac{(P - x)t_1}{\alpha} - \frac{1}{\alpha}\left(at_1 + \frac{b}{2}{t_1}^2\right) + \frac{bt_1}{\alpha^2} + \left(\frac{P - x - a}{\alpha^2} + \frac{b}{\alpha^3}\right)(e^{-\alpha t_1} - 1) \end{aligned} \qquad (12)$$

In the interval $\boldsymbol{t_1 \le t \le t_3}$,

$$A_2 = \int_{t_1}^{t_3} I_2(t)dt = \frac{W}{\alpha}\left[1 - e^{\alpha(t_1 - t_3)}\right] \qquad (13)$$

In the interval $\boldsymbol{t_1 \le t \le t_2}$,

$$\begin{aligned} A_3 &= \int_{t_1}^{t_2} I_3(t)dt \\ &= \frac{R}{\alpha}(t_2 - t_1) - \frac{1}{\alpha}\left(at_2 + \frac{b}{2}{t_2}^2\right) + \frac{1}{\alpha}\left(at_1 + \frac{b}{2}{t_1}^2\right) + \frac{b}{\alpha^2}(t_2 - t_1) \\ &\quad + \left[\frac{1}{\alpha^2}(R - a - bt_1) + \frac{b}{\alpha^3}\right]\left[e^{\alpha(t_1 - t_2)} - 1\right] \end{aligned} \qquad (14)$$

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In the interval $\boldsymbol{t_2 \le t \le t_3}$,

$$A_4 = \int_{t_2}^{t_3} I_4(t)dt$$
$$= \frac{1}{\alpha}\left(at_2 + \frac{b}{2}{t_2}^2\right) - \frac{1}{\alpha}\left(at_3 + \frac{b}{2}{t_3}^2\right) + \frac{b}{\alpha^2}(t_3 - t_2)$$
$$+ \left[\frac{1}{\alpha^2}(a + bt_3) - \frac{b}{\alpha^3}\right]\left[e^{\alpha(t_3 - t_2)} - 1\right] \quad (15)$$

In the interval $\boldsymbol{t_3 \le t \le T}$,

$$A_5 = \int_{t_3}^{T} I_5(t)dt$$
$$= \frac{1}{\alpha}\left(at_3 + \frac{b}{2}{t_3}^2\right) - \frac{1}{\alpha}\left(aT + \frac{b}{2}T^2\right) + \frac{b}{\alpha^2}(T - t_3)$$
$$+ \left[\frac{1}{\alpha^2}(a + bT) - \frac{b}{\alpha^3}\right]\left[e^{\alpha(T - t_3)} - 1\right] \quad (16)$$

Assuming that the rework process is perfect, all defective items that have undergone rework will turn into perfect items. Hence, we have the following equation:

$$xt_1 = R(t_2 - t_1)$$
$$t_2 = \frac{(R + x)t_1}{R} \quad (17)$$

The total inventory holding cost is the sum of the holding costs in both OW and RW, which is given as:

$$HC = \frac{h_1}{T}(A_1 + A_2 + A_5) + \frac{h_2}{T}(A_3 + A_4)$$
$$= \frac{h_1}{T}\left\{\frac{1}{\alpha}\left[(P - x)t_1 - \left(at_1 + \frac{b}{2}{t_1}^2\right) + \left(at_3 + \frac{b}{2}{t_3}^2\right) - \left(aT + \frac{b}{2}T^2\right) - \frac{1}{\alpha^2}(a + bt_3) + \frac{b}{\alpha^3}\right.\right.$$
$$\left.\left. - \frac{W}{\alpha}e^{\alpha(t_1 - t_3)} + \left[\frac{1}{\alpha^2}(a + bT) - \frac{b}{\alpha^3}\right]e^{\alpha(T - t_3)}\right]\right\}$$
$$+ \frac{h_2}{T}\left\{\frac{1}{\alpha}\left[R(t_2 - t_1) + \left(at_1 + \frac{b}{2}{t_1}^2\right) - \left(at_3 + \frac{b}{2}{t_3}^2\right)\right] - \frac{R}{\alpha^2}\right.$$
$$\left. + \left[\frac{1}{\alpha^2}(R - a - bt_1) + \frac{b}{\alpha^3}\right]e^{\alpha(t_1 - t_2)} + \left[\frac{1}{\alpha^2}(a + bt_3) - \frac{b}{\alpha^3}\right]e^{\alpha(t_3 - t_2)}\right\} \quad (18)$$

The total number of deteriorated items is:

$$G = \alpha(A_1 + A_2 + A_3 + A_4 + A_5)$$
$$= \left[(P - x)t_1 + R(t_2 - t_1) + W\left[1 - e^{\alpha(t_1 - t_3)}\right] - \left(aT + \frac{b}{2}T^2\right)\right]$$
$$- \frac{1}{\alpha}[P + R - x - b(t_1 - t_3)] + \left(\frac{P - x - a}{\alpha} + \frac{b}{\alpha^2}\right)e^{-\alpha t_1}$$
$$+ \left[\frac{1}{\alpha}(R - a - bt_1) + \frac{b}{\alpha^2}\right]e^{\alpha(t_1 - t_2)} + \left[\frac{1}{\alpha}(a + bt_3) - \frac{b}{\alpha^2}\right]e^{\alpha(t_3 - t_2)} + \left[\frac{1}{\alpha}(a + bT) - \frac{b}{\alpha^2}\right]e^{\alpha(T - t_3)} \quad (19)$$

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Hence, the total deteriorating cost is given by:

$$
\begin{aligned}
DC &= \frac{c_D G}{T} \\
&= \frac{c_D}{T}\Big\{\Big[(P-x)t_1 + R(t_2-t_1) + W\big[1-e^{\alpha(t_1-t_3)}\big] - \Big(aT+\frac{b}{2}T^2\Big)\Big] \\
&- \frac{1}{\alpha}[P+R-x-b(t_1-t_3)] + \Big(\frac{P-x-a}{\alpha}+\frac{b}{\alpha^2}\Big)e^{-\alpha t_1} \\
&+ \Big[\frac{1}{\alpha}(R-a-bt_1)+\frac{b}{\alpha^2}\Big]e^{\alpha(t_1-t_2)} + \Big[\frac{1}{\alpha}(a+bt_3)-\frac{b}{\alpha^2}\Big]e^{\alpha(t_3-t_2)} \\
&+ \Big[\frac{1}{\alpha}(a+bT)-\frac{b}{\alpha^2}\Big]e^{\alpha(T-t_3)}\Big\} \qquad (20)
\end{aligned}
$$

Finally, the total relevant cost per unit time, i.e., $\boldsymbol{TRC^*}$, for the model with LIFO policy is:

$$
\begin{aligned}
TRC(t_1,t_3,T) &= \frac{S}{T} + \frac{c_P P t_1}{T} + \frac{c_R x t_1}{T} \\
&+ \frac{h_1}{T}\Big\{\frac{1}{\alpha}\Big[(P-x)t_1 - \Big(at_1+\frac{b}{2}{t_1}^2\Big) + \Big(at_3+\frac{b}{2}{t_3}^2\Big) - \Big(aT+\frac{b}{2}T^2\Big)\Big] - \frac{1}{\alpha^2}(a+bt_3) + \frac{b}{\alpha^3} - \frac{W}{\alpha}e^{\alpha(t_1-t_3)} \\
&\quad + \Big[\frac{1}{\alpha^2}(a+bT)-\frac{b}{\alpha^3}\Big]e^{\alpha(T-t_3)}\Big\} \\
&+ \frac{h_2}{T}\Big\{\frac{1}{\alpha}\Big[R(t_2-t_1) + \Big(at_1+\frac{b}{2}{t_1}^2\Big) - \Big(at_3+\frac{b}{2}{t_3}^2\Big)\Big] - \frac{R}{\alpha^2} + \Big[\frac{1}{\alpha^2}(R-a-bt_1)+\frac{b}{\alpha^3}\Big]e^{\alpha(t_1-t_2)} \\
&\quad + \Big[\frac{1}{\alpha^2}(a+bt_3)-\frac{b}{\alpha^3}\Big]e^{\alpha(t_3-t_2)}\Big\} \\
&+ \frac{c_D}{T}\Big\{\Big[(P-x)t_1 + R(t_2-t_1) + W\big[1-e^{\alpha(t_1-t_3)}\big] - \Big(aT+\frac{b}{2}T^2\Big)\Big] - \frac{1}{\alpha}[P+R-x-b(t_1-t_3)] \\
&\quad + \Big(\frac{P-x-a}{\alpha}+\frac{b}{\alpha^2}\Big)e^{-\alpha t_1} + \Big[\frac{1}{\alpha}(R-a-bt_1)+\frac{b}{\alpha^2}\Big]e^{\alpha(t_1-t_2)} + \Big[\frac{1}{\alpha}(a+bt_3)-\frac{b}{\alpha^2}\Big]e^{\alpha(t_3-t_2)} \\
&\quad + \Big[\frac{1}{\alpha}(a+bT)-\frac{b}{\alpha^2}\Big]e^{\alpha(T-t_3)}\Big\} \qquad (21)
\end{aligned}
$$

Referring to equation (21), we note that finding the optimal values of $\boldsymbol{t_1}$, $\boldsymbol{t_3}$, and $\boldsymbol{T}$ analytically is extremely tedious. Hence, we have explored alternative methods and obtained the best solution for $\boldsymbol{TRC}$ numerically, which will be discussed in Section 2.5.

2.4 FIFO Policy

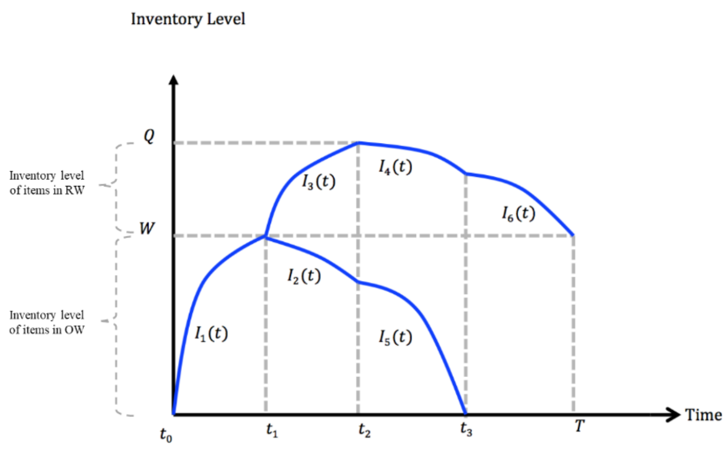

Figure 2. Inventory level with FIFO policy

The FIFO policy is a phenomenon opposite to the LIFO policy. In this system, items in OW which are stored first will be completely utilised first. Figure 2 illustrates the production system of the two-warehouse model with FIFO policy.

Similarly, the production cycle begins at $\boldsymbol{t_0}$ where deterioration and fulfilment of demand occurs simultaneously in the first interval. Since the production process is assumed to be imperfect, defective items are produced in this interval. The defectives are

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separated and kept aside to be sent to RW at $\boldsymbol{t_1}$. The differential equation representing the change of inventory level in OW during the interval $\boldsymbol{t_0 \le t \le t_1}$ is denoted by:

$$\frac{dI_1(t)}{dt} = P - x - f(t) - \alpha I_1(t) \qquad ; \quad t_0 \le t \le t_1 \qquad (22)$$

By solving differential equation (22) with the initial condition, $\boldsymbol{I_1(t_0) = 0}$, the inventory level in this interval is given as:

$$I_1(t) = \left[\frac{(P - x - a)}{\alpha} + \frac{b}{\alpha^2}\right](1 - e^{-\alpha t}) - \frac{bt}{\alpha} \qquad (23)$$

While defective items were sent to RW to undergo rework process, items in OW are kept idle or on standby during the interval $\boldsymbol{t_1 \le t \le t_2}$. However, due to deterioration, items will deplete at the rate $\boldsymbol{\alpha}$ in which the change of inventory level is represented by:

$$\frac{dI_2(t)}{dt} = -\alpha I_2(\mathrm{t}) \qquad ; \quad t_1 \le t \le t_2 \qquad (24)$$

Considering the boundary condition $\boldsymbol{I_2(t_1) = W}$, where $\boldsymbol{W}$ represents the maximum capacity of OW, we have the following inventory level:

$$I_2(t) = We^{\alpha(t_1 - t)} \qquad (25)$$

The rework process in RW begins in the interval $\boldsymbol{t_1 \le t \le t_2}$, in which items are depleted due to deterioration and fulfilment of demand. The change of inventory level during this interval is represented by:

$$\frac{dI_3(t)}{dt} = R - f(t) - \alpha I_3(\mathrm{t}) \qquad ; \quad t_1 \le t \le t_2 \qquad (26)$$

Considering the boundary condition $\boldsymbol{I_3(t_1) = 0}$, the inventory level in this interval is denoted by:

$$I_3(t) = \frac{1}{\alpha}(R - a - bt) + \frac{b}{\alpha^2} - \left[\frac{1}{\alpha}(R - a - bt_1) + \frac{b}{\alpha^2}\right]e^{\alpha(t_1 - t)} \qquad (27)$$

The rework process in RW ends at $\boldsymbol{t_2}$ where these items will be on standby while demands are fulfilled by items in OW during the interval $\boldsymbol{t_2 \le t \le t_3}$. However, items in RW will continue to deplete due to deterioration only. The change of inventory level during this interval is denoted by:

$$\frac{dI_4(t)}{dt} = -\alpha I_4(\mathrm{t}) \qquad ; \quad t_2 \le t \le t_3 \qquad (28)$$

Considering the boundary condition $\boldsymbol{I_4(t_2) = Q}$, the inventory level in this interval is denoted by:

$$I_4(t) = Qe^{\alpha(t_2 - t)} \qquad (29)$$

Simultaneously, items stored in OW will begin depleting to fulfil demand in the interval $\boldsymbol{t_2 \le t \le t_3}$ until they are fully depleted at $\boldsymbol{t_3}$. The change of inventory level in this interval is represented by:

$$\frac{dI_5(t)}{dt} = -f(t) - \alpha I_5(\mathrm{t}) \qquad ; \quad t_2 \le t \le t_3 \qquad (30)$$

Considering the boundary condition $\boldsymbol{I_5(t_3) = 0}$, the inventory level in this interval is denoted by:

$$I_5(t) = \left[\frac{1}{\alpha}(a + bt_3) - \frac{b}{\alpha^2}\right]e^{\alpha(t_3 - t)} - \frac{(a + bt)}{\alpha} + \frac{b}{\alpha^2} \qquad (31)$$

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Once all items in OW are fully depleted at $\boldsymbol{t_3}$, demand will be fulfilled by the remaining inventory in RW during the last interval, while deterioration occurs simultaneously. The production cycle ends at $\boldsymbol{T}$, in which the inventory will be fully depleted. The change in the inventory level is denoted by:

$$\frac{dI_6(t)}{dt} = -f(t) - \alpha I_6(\mathrm{t}) \qquad ; \quad t_3 \le t \le T \tag{32}$$

Considering the boundary condition $\boldsymbol{I_6(T) = 0}$, the inventory level in this interval is denoted by:

$$I_6(t) = \left[\frac{(a+bT)}{\alpha} - \frac{b}{\alpha^2}\right] e^{\alpha(T-t)} - \frac{(a+bt)}{\alpha} + \frac{b}{\alpha^2} \tag{33}$$

The maximum inventory of OW, $\boldsymbol{W}$, is governed by equation $\boldsymbol{W = I_1(t_1)}$. Hence, we have:

$$W = \left[\frac{(P-x-a)}{\alpha} + \frac{b}{\alpha^2}\right](1 - e^{-\alpha t_1}) - \frac{bt_1}{\alpha} \tag{34}$$

and the maximum inventory of RW, $\boldsymbol{Q}$, is governed by equation $\boldsymbol{Q = I_3(t_2)}$, where:

$$Q = \frac{1}{\alpha}(R - a - bt_2) + \frac{b}{\alpha^2} - \left[\frac{1}{\alpha}(R - a - bt_1) + \frac{b}{\alpha^2}\right] e^{\alpha(t_1 - t_2)} \tag{35}$$

Next, let $\boldsymbol{A_i}$ denote the time-weighted inventory for each interval, where $\boldsymbol{i = 1, 2, \dots, 6}$, and it represents the amount of inventory under the curve $\boldsymbol{I_i(t)}$. The following are the respective equations for $\boldsymbol{A_i}$ in the corresponding intervals:

In the interval $\boldsymbol{t_0 \le t \le t_1}$,

$$\begin{aligned} A_1 &= \int_0^{t_1} I_1(t)\,dt \\ &= \left[\frac{(P-x)}{\alpha} + \frac{b}{\alpha^2}\right] t_1 - \frac{1}{\alpha}\left(at_1 + \frac{b}{2}{t_1}^2\right) + \left(\frac{P-x-a}{\alpha^2} + \frac{b}{\alpha^3}\right)(e^{-\alpha t_1} - 1) \end{aligned} \tag{36}$$

In the interval $\boldsymbol{t_1 \le t \le t_2}$, for OW and RW, respectively

$$A_2 = \int_{t_1}^{t_3} I_2(t)dt = \frac{W}{\alpha}\left[1 - e^{\alpha(t_1 - t_2)}\right] \tag{37}$$

$$\begin{aligned} A_3 &= \int_{t_1}^{t_2} I_3(t)dt \\ &= \left[\frac{R}{\alpha} + \frac{b}{\alpha^2}\right](t_2 - t_1) + \frac{1}{\alpha}\left[\left(at_1 + \frac{b}{2}{t_1}^2\right) - \left(at_2 + \frac{b}{2}{t_2}^2\right)\right] \\ &+ \left[\frac{1}{\alpha^2}(R - a - bt_1) + \frac{b}{\alpha^3}\right]\left[e^{\alpha(t_1 - t_2)} - 1\right] \end{aligned} \tag{38}$$

In the interval $\boldsymbol{t_2 \le t \le t_3}$, for OW and RW, respectively

$$A_4 = \int_{t_2}^{t_3} I_4(t)dt = \frac{Q}{\alpha}\left[1 - e^{\alpha(t_2 - t_3)}\right] \tag{39}$$

$$\begin{aligned} A_5 &= \int_{t_2}^{t_3} I_5(t)dt \\ &= \frac{b}{\alpha^2}(t_3 - t_2) + \frac{1}{\alpha}\left[\left(at_2 + \frac{b}{2}{t_2}^2\right) - \left(at_3 + \frac{b}{2}{t_3}^2\right)\right] \\ &+ \left[\frac{1}{\alpha^2}(a + bt_3) - \frac{b}{\alpha^3}\right]\left[e^{\alpha(t_3 - t_2)} - 1\right] \end{aligned} \tag{40}$$

In the interval $\boldsymbol{t_3 \le t \le T}$,

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$$\begin{aligned}A_6 &= \int_{t_3}^{T} I_6(t)dt\\ &= \frac{b}{\alpha^2}(T-t_3) + \frac{1}{\alpha}\left[\left(at_3 + \frac{b}{2}{t_3}^2\right) - \left(aT + \frac{b}{2}T^2\right)\right]\\ &+ \left[\frac{1}{\alpha^2}(a+bT) - \frac{b}{\alpha^3}\right]\left[e^{\alpha(T-t_3)} - 1\right]\end{aligned} \tag{41}$$

Similar to LIFO policy, we have:

$$\begin{aligned} xt_1 &= R(t_2 - t_1)\\ t_2 &= \frac{(R+x)t_1}{R}\end{aligned} \tag{42}$$

The total inventory holding cost is the sum of the holding costs in both OW and RW, which is given as:

$$\begin{aligned}HC &= \frac{h_1}{T}(A_1 + A_2 + A_5) + \frac{h_2}{T}(A_3 + A_4 + A_6)\\ &= \frac{h_1}{T}\left\{\frac{1}{\alpha}\left[(P-x)t_1 - \left(at_1 + \frac{b}{2}{t_1}^2\right) - \left(at_3 + \frac{b}{2}{t_3}^2\right) + \left(at_2 + \frac{b}{2}{t_2}^2\right) - \frac{1}{\alpha^2}(a+bt_2)\right.\right.\\ &\left.\left.+ \frac{b}{\alpha^3} - \frac{W}{\alpha}e^{\alpha(t_1-t_2)} + \left[\frac{1}{\alpha^2}(a+bt_3) - \frac{b}{\alpha^3}\right]e^{\alpha(t_3-t_2)}\right]\right\}\\ &+ \frac{h_2}{T}\left\{\frac{1}{\alpha}\left[R(t_2-t_1) + \left(at_1 + \frac{b}{2}{t_1}^2\right) - \left(at_2 + \frac{b}{2}{t_2}^2\right) + \left(at_3 + \frac{b}{2}{t_3}^2\right) - \left(aT + \frac{b}{2}T^2\right)\right]\right.\\ &\left.- \frac{1}{\alpha^2}(a+bt_3) + \frac{b}{\alpha^3} - \frac{Q}{\alpha}e^{\alpha(t_2-t_3)} + \left[\frac{1}{\alpha^2}(a+bT) - \frac{b}{\alpha^3}\right]e^{\alpha(T-t_3)}\right\}\end{aligned} \tag{43}$$

The deteriorating cost per unit time is given by:

$$\begin{aligned}DC &= \frac{c_D\alpha}{T}(A_1 + A_2 + A_3 + A_4 + A_5 + A_6)\\ &= \frac{c_D}{T}\left\{Pt_1 - \left(aT + \frac{b}{2}T^2\right) - \frac{1}{\alpha}(a+bt_2) - \frac{1}{\alpha}(a+bt_3) + \frac{2b}{\alpha^2} - We^{\alpha(t_1-t_2)} - Qe^{\alpha(t_2-t_3)}\right.\\ &\left.+ \left[\frac{1}{\alpha}(a+bt_3) - \frac{b}{\alpha^2}\right]e^{\alpha(t_3-t_2)} + \left[\frac{1}{\alpha}(a+bT) - \frac{b}{\alpha^2}\right]e^{\alpha(T-t_3)}\right\}\end{aligned} \tag{44}$$

Finally, $\boldsymbol{TRC}^*$ for FIFO policy is:

$$\begin{aligned}TRC(t_1,t_3,T) &= \frac{S}{T} + \frac{c_P P t_1}{T} + \frac{c_R x t_1}{T} + \frac{h_1}{T}\left\{\frac{1}{\alpha}\left[(P-x)t_1 - \left(at_1 + \frac{b}{2}{t_1}^2\right) + \left(at_2 + \frac{b}{2}{t_2}^2\right) - \left(at_3 + \frac{b}{2}{t_3}^2\right)\right]\right.\\ &\left.- \frac{1}{\alpha^2}(a+bt_2) + \frac{b}{\alpha^3} - \frac{W}{\alpha}e^{\alpha(t_1-t_2)} + \left[\frac{1}{\alpha^2}(a+bt_3) - \frac{b}{\alpha^3}\right]e^{\alpha(t_3-t_2)}\right\}\\ &+ \frac{h_2}{T}\left\{\frac{1}{\alpha}\left[R(t_2-t_1) + \left(at_1 + \frac{b}{2}{t_1}^2\right) - \left(at_2 + \frac{b}{2}{t_2}^2\right) + \left(at_3 + \frac{b}{2}{t_3}^2\right)\right.\right.\\ &\left.\left.- \left(aT + \frac{b}{2}T^2\right)\right] - \frac{1}{\alpha^2}(a+bt_3) + \frac{b}{\alpha^3} - \frac{Q}{\alpha}e^{\alpha(t_2-t_3)} + \left[\frac{1}{\alpha^2}(a+bT) - \frac{b}{\alpha^3}\right]e^{\alpha(T-t_3)}\right\}\\ &+ \frac{c_D}{T}\left\{Pt_1 - \left(aT + \frac{b}{2}T^2\right) - \frac{1}{\alpha}(a+bt_2) - \frac{1}{\alpha}(a+bt_3) + \frac{2b}{\alpha^2} - We^{\alpha(t_1-t_2)}\right.\\ &\left.- Qe^{\alpha(t_2-t_3)} + \left[\frac{1}{\alpha}(a+bt_3) - \frac{b}{\alpha^2}\right]e^{\alpha(t_3-t_2)} + \left[\frac{1}{\alpha}(a+bT) - \frac{b}{\alpha^2}\right]e^{\alpha(T-t_3)}\right\}\end{aligned} \tag{45}$$

We have utilised the same approach as in Section 2.3 in which the optimal solution of $\boldsymbol{TRC(t_1, t_3, T)}$ is obtained numerically as discussed in the next section.

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2.5 Solution Procedure

Numerical algorithms for constrained nonlinear optimisation can be widely categorised into gradient-based methods and direct search methods. The first derivatives (gradients) or second derivatives (Hessians) information is used in the gradient search methods, while derivative information is not used in direct search method (Wolfram, 2020).

$\boldsymbol{TRC}^*$ is a non-polynomial equation, where its second derivative with respect to $\boldsymbol{t_1}$, $\boldsymbol{t_3}$, and $\boldsymbol{T}$ is hard to differentiate. Hence, a closed-form solution could not be derived and an optimal solution cannot be guaranteed. In this study, generalised reduced gradient (GRG) has been chosen as the solving method. Hence, the Microsoft Excel Solver is used as a solution tool. GRG converts the constrained problem into an unconstrained one. The extended reduced gradient method, known as the GRG method, accommodates nonlinear inequality constraints. Using this method, a search direction is found in which the current active constraints remain precisely active for any small move.

The following algorithm is used:

1. Set $\boldsymbol{t_0 = 0}$.
2. Determine the values of $\boldsymbol{t_1}$, $\boldsymbol{t_3}$, and $\boldsymbol{T}$, which satisfy the following constraints:
   For LIFO,
   $\boldsymbol{I_1(t_0) = 0, I_1(t_1) = I_2(t_1), I_3(t_1) = 0, I_4(t_3) = 0, I_2(t_3) = I_5(t_3)}$ and $\boldsymbol{I_5(T) = 0}$

   For FIFO,
   $\boldsymbol{I_1(t_0) = 0, I_1(t_1) = I_2(t_1), I_3(t_1) = 0, I_4(t_3) = I_6(t_3), I_2(t_2) = I_5(t_2), I_5(t_3) = 0}$ and $\boldsymbol{I_6(T) = 0}$
3. Compute $\boldsymbol{t_2} = \frac{\boldsymbol{(R+x)t_1}}{\boldsymbol{R}}$.
4. Compute $\boldsymbol{TRC}^*$ using equations (21) and (45) for LIFO and FIFO respectively.

Aside from the GRG method, we have utilised the Wolfram Language function, which solves for numeric local constrained optimisation, which is known as the FindMinimum function. This function uses the interior point methods to find the solution to problems with constraints (Wolfram, 2020). We have utilised the built-in function to verify our results and we note that both the Microsoft Excel Solver and Mathematica software, provide the same results.

2.6 Numerical Example and Sensitivity Analysis

The following numerical example has been considered to provide an illustration of the proposed policies in this study. The parameters used in the model are $\boldsymbol{P = 3000}$, $\boldsymbol{R = 1000}$, $\boldsymbol{x = 500}$, $\boldsymbol{\alpha = 0.04}$, $\boldsymbol{a = 550}$, and $\boldsymbol{b = 200}$. The costs involved in this model are given as follows, $\boldsymbol{S} = \$\boldsymbol{1000}$, $\boldsymbol{c_P} = \$\boldsymbol{2.00}$, $\boldsymbol{c_R} = \$\boldsymbol{3.00}$, $\boldsymbol{d} = \$\boldsymbol{2.50}$, $\boldsymbol{h_1} = \$\boldsymbol{1.50}$, and $\boldsymbol{h_2} = \$\boldsymbol{2.50}$.

Based on the parameters considered, the optimal solution of $\boldsymbol{TRC}^*$ for the LIFO policy is \$3047.39, and it is achieved at $\boldsymbol{t_1^* = 0.2556}$, $\boldsymbol{t_3^* = 0.4609}$, and $\boldsymbol{T^* = 1.1354}$ (correct to four decimal places).

On the other hand, using the same parameters as LIFO policy, the optimal solution of $\boldsymbol{TRC}^*$ for the FIFO policy is \$3076.34, which is achieved at $\boldsymbol{t_1^* = 0.2516}$, $\boldsymbol{t_3^* = 1.0588}$, and $\boldsymbol{T^* = 1.1203}$ (correct to four decimal places).

Table 1 shows the changes in $\boldsymbol{TRC}^*$ as the parameters are reduced and increased by 25% in both the LIFO and FIFO systems. The changes in the total number of items produced, defective items, demand, and deteriorated items are presented in Table 2.

We note that the results obtained are similar in both systems in which the value of $\boldsymbol{TRC}^*$ increases as the parameters increased. This behaviour is true for all parameters except when the value of $\boldsymbol{P}$ is increased. Since the increment in $\boldsymbol{P}$ shall mean that the items are produced at a faster rate in a shorter period, we note that fewer items are being produced. In return, fewer number of defective items are produced in the system. The decrement of the processing, rework processing, and holding costs in RW result in the decrement of $\boldsymbol{TRC}^*$.

Table 1. Comparison of the difference in the TRC^* under varying parameters.

Parameters	−25%, Optimal, +25%	LIFO				FIFO			
		$\boldsymbol{TRC}^*$	$\boldsymbol{t_1^*}$	$\boldsymbol{t_3^*}$	$\boldsymbol{T^*}$	$\boldsymbol{TRC}^*$	$\boldsymbol{t_1^*}$	$\boldsymbol{t_3^*}$	$\boldsymbol{T^*}$
	2250	3083.67	0.36	0.63	1.19	3113.57	0.35	1.09	1.17
$\boldsymbol{P}$	3000	3047.39	0.26	0.46	1.14	3076.34	0.25	1.06	1.12
	3750	3020.29	0.20	0.36	1.11	3046.72	0.20	1.05	1.10
	750	3040.90	0.26	0.46	1.14	3054.10	0.26	1.11	1.14
$\boldsymbol{R}$	1000	3047.39	0.26	0.46	1.14	3076.34	0.25	1.06	1.12
	1250	3051.26	0.25	0.46	1.13	3089.49	0.25	1.03	1.11
	375	2972.45	0.26	0.41	1.14	2995.95	0.25	1.08	1.12
$\boldsymbol{x}$	500	3047.39	0.26	0.46	1.14	3076.34	0.25	1.06	1.12
	625	3121.25	0.26	0.51	1.14	3154.57	0.25	1.04	1.12
$\boldsymbol{\alpha}$	0.03	3031.63	0.26	0.46	1.15	3060.93	0.25	1.07	1.13
	0.04	3047.39	0.26	0.46	1.14	3076.34	0.25	1.06	1.12

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	0.05	3063.01	0.25	0.46	1.12	3091.63	0.25	1.05	1.11
	412.5	2637.03	0.21	0.43	1.17	2672.17	0.21	1.06	1.15
a	550.0	3047.39	0.26	0.46	1.14	3076.34	0.25	1.06	1.12
	687.5	3440.62	0.30	0.50	1.11	3460.15	0.30	1.07	1.11
	150	2951.18	0.26	0.48	1.21	2982.44	0.26	1.12	1.19
b	200	3047.39	0.26	0.46	1.14	3076.34	0.25	1.06	1.12
	250	3136.46	0.25	0.45	1.08	3163.42	0.25	1.01	1.06

Table 2. Analysis of change in various parameters on the total inventory items.

Parameters	−25%, Optimal, +25%	LIFO				FIFO			
		Total Items Produced	Total Defectives	Total Demand	Total Deteriorated Items	Total Items Produced	Total Defectives	Total Demand	Total Deteriorated Items
	2250	807.82	179.51	795.24	12.57	795.75	176.83	783.51	12.24
P	3000	766.82	127.80	753.37	13.45	754.71	125.78	741.65	13.06
	3750	747.80	99.71	733.92	13.88	736.22	98.16	722.73	13.49
	750	771.66	128.61	758.16	13.49	768.02	128.00	754.64	13.38
R	1000	766.82	127.80	753.37	13.45	754.71	125.78	741.65	13.06
	1250	764.38	127.40	750.95	13.43	747.10	124.52	734.23	12.88
	375	768.19	96.02	754.40	13.79	758.15	94.77	744.70	13.46
x	500	766.82	127.80	753.37	13.45	754.71	125.78	741.65	13.06
	625	766.22	159.63	753.13	13.09	752.52	156.78	739.85	12.67
	0.03	772.56	128.76	762.27	10.29	760.10	126.68	750.11	9.99
α	0.04	766.82	127.80	753.37	13.45	754.71	125.78	741.65	13.06
	0.05	760.73	126.79	744.27	16.46	749.47	124.91	733.46	16.02
	412.5	629.74	104.96	617.34	12.40	615.37	102.56	603.48	11.89
a	550.0	766.82	127.80	753.37	13.45	754.71	125.78	741.65	13.06
	687.5	904.79	150.80	890.55	14.24	897.59	149.60	883.56	14.03
	150	788.97	131.50	774.26	14.71	774.37	129.06	760.16	14.21
b	200	766.82	127.80	753.37	13.45	754.71	125.78	741.65	13.06
	250	748.97	124.83	736.52	12.45	739.00	123.17	726.86	12.15

2.7 Comparison between LIFO and FIFO policies

In general, the TRC^* of the LIFO system is lower than the TRC^* of the FIFO system, where $TRC^*_{LIFO} = \$3047.39 < TRC^*_{FIFO} = \$3076.34$. Based on the sensitivity analysis of both policies, the following are the features that we have identified.

We observed that changes in the value of t_1 affect the number of total produced items and defective items, while changes in the value of T affect the total demand in the system. Note that in FIFO policy, items that are stored in OW will be utilised or distributed first, followed by items in RW. Hence, items are stored longer in RW, which in turn results in a higher holding cost.

Referring to the optimal solution presented, we can see that TRC^* for the FIFO policy is higher than the TRC^* in the LIFO policy. Hence, we can conclude that given the same value of parameters, the LIFO system has a lower TRC^*.

3. Conclusion

3.1 Conclusion and Further Research

The total relevant cost, TRC^*, for both policies is a nonlinear equation, where its second derivative with respect to t_1, t_3, and T is complicated. Hence, we have utilised the optimisation tools in Microsoft Excel Solver and Mathematica to obtain an optimal solution for the proposed

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model. According to the results obtained, we observed that the equation for TRC^* is convex at the optimal values of t_1, t_3, and T. In other words, the optimal solution of TRC^* is minimum at the aforementioned optimal times. A sensitivity analysis was conducted for both policies to provide illustration on the derived results. In addition, we also note that several studies such as Sett et al. (2012) and Lee (2006) to name a few, derived similar results as obtained in this paper.

Several limitations were identified while conducting the study. First, the assumption that deterioration rate is constant is unrealistic. Deterioration rates may be affected by environmental factors and workmanship of the items produced. The type of deterioration rates may be further explored as different storage space may have different facilities, which may result in the difference in the deterioration rate. Next, shortages and backlog are also commonly present in the market when demand is higher than supply. Hence, exploring this factor further would be beneficial in planning the right number of items to be produced to ensure fulfilment of demand.

3.2 Managerial Insights

The proposed model in this paper may provide managerial insights to aid manufacturers in optimising the total cost of their production system when managing two storage facilities involving deteriorating items.

Our approach in incorporating a linearly increasing demand rate would allow inventory operators to plan their production accordingly when launching new items into the market. We observe that a newly launched product such as cosmetics, fashion items, and mobile phones experience a linearly increasing demand rate for a certain period of time upon being introduced into the market.

Furthermore, introducing an imperfect production process in the manufacturing system is a more realistic condition. In some instances, rework process is a more cost-efficient approach rather than producing scrap and disposing defective items. Besides, we have assumed that the rework process is only carried out in RW. This would be beneficial to manufacturers with limited number of machines as they are able to focus solely on production process first, prior to repairing the defectives.

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