METHODS IN SOLVING RATIO AND PROPORTION PROBLEMS AMONG YEAR FIVE PUPILS

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Abstract: Basic concepts of ratio and proportion are essential especially in solving problems in primary mathematics. This study aimed to identify seven Year Five pupils' methods of solving ratio and proportion problems and was based on radical constructivism using case studies. The pupils were posed with word problems and using the clinical interview technique, data was collected through pupils' visualization, verbal explanations, and written responses. The findings showed that the children used experience and previous ideas to reflect their thinking on ratio and proportion. Both procedural and conceptual methods were utilized by the year five pupils in solving the problems. The procedural methods consist of part-to-part and part-to-whole comparisons in stating the ratios. Equivalent ratios, simplified ratios and equivalent fractions were the conceptual methods used in solving the problems. It was identified that the pupils used the multiplicative relationship to interrelate ratios with proportions which will be the key in learning ratio and proportions.

Keywords: *Clinical Interview, Problem Solving Methods, Ratio And Proportion, Radical Constructivism*

INTRODUCTION

Ratio and proportion are important in daily mathematics. Research shows that ratio and proportion are important topics in the mathematics curriculum (Artut & Pelen, 2015; Andini & Jupri, 2017; Cramer, 2017; Diba & Prabawanto, 2019; Dole, 2008; Dougherty et al., 2016; Hulbert et al., 2017; Lobato et al., 2014; Tiflis et al., 2019; Petit et al., 2020). In the Malaysian mathematics curriculum, pupils must solve word problems related to ratio and proportion. There are a few important elements that help in acquiring the knowledge of ratio and proportion. An intuitive understanding of proportional relations supports the development of proportional thinking (Sumarto et al., 2014). Children should think multiplicatively (relatively) and not additively (absolutely) while working with ratio and proportion problems. Multiplication is sometimes conceptualized as repeated addition by primary pupils, and they use this knowledge to solve multiplication word problems. Besides, Larsson, Petterson & Andrews (2017) stated that primary pupils conceptualized multiplication as repeated addition or equal groups.

Noor Fazura, Sharifah & Leong (2017) also believed that students must know the basic concepts of ratio and proportion to reason proportionally. Further, what is important is the ability of the brain to connect mathematics with real-life situations in conjunction to make meaning of the world. Further, the importance of prior knowledge of multiplication, division and the representation of fractions help in relating ratios to proportions (Fuson, 2005; Battista, 2012; Hulbert et al., 2017). The methods used by the pupils are based on their knowledge. Debrenti (2015) believes that students should solve problems using their problem-solving skills. Further Abrahamson (2003) assumes that exposing verbal problems to students, can be a way to make students understand the proportion. In this way, they will be able to construct their knowledge.

Silver (1986) states that problem-solving is a complex knowledge domain that involves the application of both procedural and conceptual knowledge. Procedural knowledge is known as action sequences for solving problems (Rittle-Johnson & Alibali Wagner, 1999) which includes facts skills, procedures, algorithms, or methods. Rittle-Johnson & Alibali Wagner (1999) describes conceptual knowledge as explicit or implicit understanding and interrelations between pieces of knowledge in a domain. It is also known as "ideas, relationships, connections, or having a 'sense of something (Barr, Doyle et al., 2003).

In this study, the problem-solving approach used is strongly based on constructivist learning theory. Nik Azis (2014) refers to problem-solving as a method used by the respondents to overcome the disequilibrium faced by the pupils. Consequently, as this study is a qualitative study on the radical constructivist paradigm, therefore, the definition of problem-solving is referred to as a conflict that a pupil goes through when he or she fails to assimilate a task given. In this research, problem-solving refers to the methods used by the pupils to overcome the disequilibrium which occurs when the pupils attempt to respond to the task given to them.

This study aimed to identify seven Year Five pupils' methods of solving ratio and proportion problems. Their conceptions will be identified through their abstractions or generalizations from their experiences.

METHODOLOGY

Research Design

This qualitative research utilizes the case study as the research design, whereas the clinical interview is used as the technique of data collection. Cobb and Steffe (1983) also claim that the clinical interview helps to investigate the sequence procedures taken by students in building their mathematics concept. Nik Azis (2014) and Sharifah Norul Akmar (1997) claims that clinical interview helps in answering research questions on students' conceptions because the depth detailed information can be gathered from the children's thinking and their ability to build their schemes through interaction with the surrounding and environment.

The data analysis involved four stages, namely transcription of the interview recording into the writing, the establishment of case studies involving descriptions of the behavior of the research participants about ideas of the ratio and proportion, analysis and cross-case analysis of the participants through triangulation method using 'verbal and non-verbal responses, and the identification of the year five pupils' methods used in solving ratio and proportion problems. The data collected through clinical interview consists of children's verbal, written work, and their changes in attitude nonverbally while they work on the problem solving.

Participants

The samples chosen in this study are three boys and four girls of year five pupils from the same school and the samples are purposefully selected cases. Out of the seven respondents, three of the respondents are high achievers, two are moderate in their achievement, and two of the respondents are low achievers in the mathematics test conducted at the school concerned. The purposive sampling is being utilized in this study to discover and gain information in depth and rich information (Patton, 2002). Further it helped in gathering the information of the methods used by them to solve ratio and proportion word problems.

Instruments

The fourth interview session of this study was on the methods used by the year five pupils to solve basic word problems of ratio and proportion. Six problems were posed to the respondents. The problems were adapted from worksheets on ratio and proportion from foreign universities. They wrote as many ratios as possible that they can form and determine whether they are part-to-part, part-to-whole or whole to part ratios. Further, the solutions and responses given by the pupils are the indications of their conceptions of ratio and proportion. The word problems involved are as below: -

1. **Problem 1**. Someone surveyed pupils taking breakfast in the morning. Four out of five pupils take breakfast in the morning. How many pupils were surveyed? Only five? If five pupils were surveyed, how many do not take breakfast in the morning? If they survey 10 pupils, how many of the pupils take their breakfast in the morning?

2. **Problem 2**. 300 children participated in a painting contest. 120 of them were boys.
   a. What is the ratio of the number of boys to the number of girls?
   b. What is the ratio of the number of girls to the number of boys?
   c. What is the ratio of the number of boys to the total number of children?
   d. What is the ratio of the number of girls to the total number of children?

3. **Problem 3**. 6 students ride the bus in a class of 28 students. All the other students walk to school.
   a. What is the ratio of students who ride the bus to those who walk?
   b. What is the ratio of walkers to students in the class?
   c. If the school of 336 has the same ratio of walkers to riders, how many students in the school ride the bus?

4. **Problem 4**. Suppose a bag of candy contains 24 grapes flavoured, 18 strawberry flavoured, and 3 chocolate flavoured pieces.

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a. One ratio, for grape to strawberry would be 24:18. Find two more ratios that could compare the grape and strawberry candies in this bag.
b. Find two different ratios for comparing the number of chocolate pieces to the total pieces of candy.

5. **Problem 5**. In a carton of eggs containing 5 brown and 7 white eggs, show as many ratios as possible that can be formed from this situation.

6. **Problem 6.** The standard five pupils are planning to sell cups of milo drink at the canteen day this Sunday. If 2 spoonfuls of milo are needed for one cup of milo drink. How many spoonfuls of milo will be needed to make 16 cups of milo?

RESULTS AND DISCUSSION

The year five pupils solved the ratio and proportion problems using procedural and conceptual methods.

Procedural Methods

Almost all the year five pupils solved ratio and proportion problems procedurally by comparing numbers or quantities. For example, the items and the quantities are counted and compared as numbers. Further, in Problem 1 four respondents initially had the idea that the total number of pupils as two parts. The part which represents the pupils who took breakfast is part *a*, and the other part which represents the pupils who did not take breakfast is part *b*. In this situation, they used subtraction to get the number of pupils who did not take breakfast [b=(a+b) –a]. Four of the respondents used the minus operation and one of them used concrete drawings. The respondent crossed out the one pupil who did not take the breakfast. The ticks and cross marks in Diagram 1 are methods she had used to solve the problem.

... Because, in the box, it is stated four out of five pupils…by right five pupils must take breakfast…but only four took breakfast. So, five minus four I will get one.

.(R2, 23.12.2017,16)

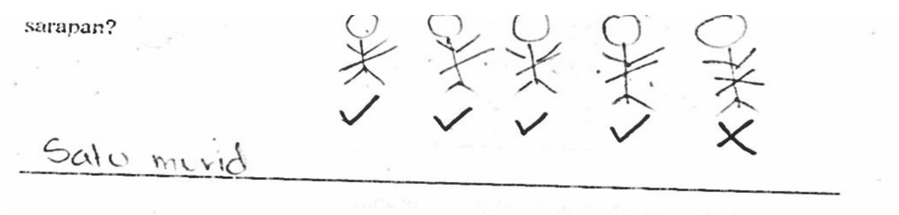

Diagram 1. Ticks and Crosses Method

In addition to that, respondents had the idea that the addition of both the parts as (a +b) in Problem 2. For example, they added the parts (2parts and 3 parts) and they concluded that the whole is equal to five parts. To find the parts of a whole, the respondents used subtraction to minus the parts from the whole. They also had the idea that the whole is more than the parts. In one situation the respondents have the assumption that the number of boys as part a, the number of girls as part b and the total number of children as the total parts of a and b (a + b) as shown in Diagram 2. Further, the word *'out of'* was utilized to show the concept of the whole.

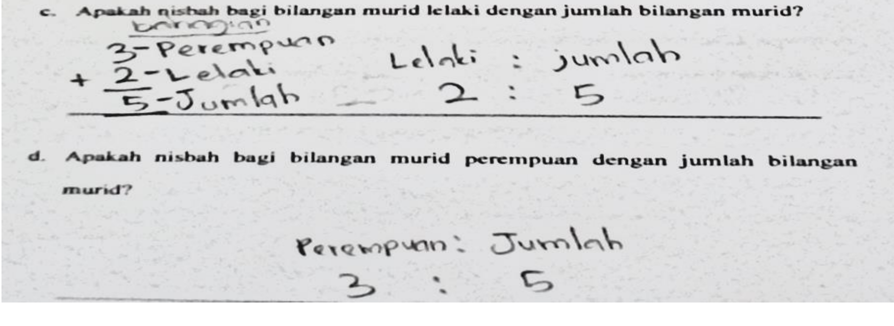

Diagram 2. Solving with part to Whole Method

In Problem 3, all the respondents assumed the whole as twenty-eight pupils (a +b). The number of pupils who took the bus was six of them (part a). Therefore, one part of the whole was described. To obtain the other part, the respondents have subtracted six from twenty-eight to obtain twenty-two. The respondents obtained twenty-two as the number of pupils who walk to school (part b). Further, all the respondents compared the number of pupils who took the bus to those who walk as a comparison of quantities a:(a+b) −a and stated the ratio as six to twenty-two as shown in Diagram 3.

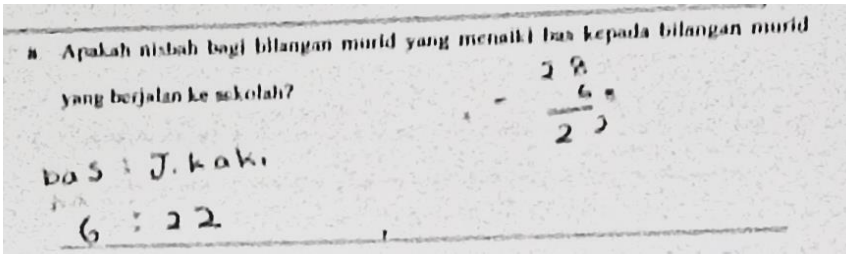

Diagram 3. Solving with part-to-part Ratios

All the respondents could compare part-to-part and state the ratios in duals (a: b and b: a). Almost all the respondents could compare part-to-whole (out of) a: a+b or b: a+b and a few could compare whole to part (a+b: a or a+b: b) as in Diagram 4. Here they were able to see the relation between the parts and whole. The first respondent described whole to part as below.

… I can also start comparing it from the total… I can compare all the eggs with the white eggs and that will give ratio twelve to seven …and the comparison of all the eggs to the brown eggs will be ratio twelve to five.

(R1,6.1.2018,28)

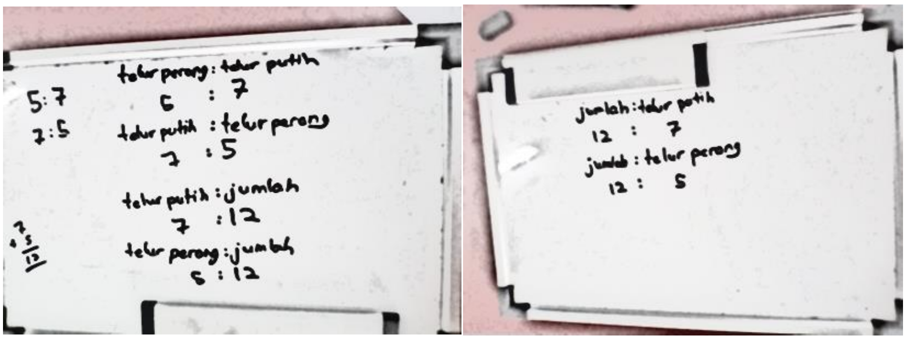

Diagram 4. Solving with part-to-part, part-to-whole and whole-to-part ratios.

However, one respondent could state ratios by comparing them with non-existing objects. He could form ratios comparing eggs with non-existing colours. (0: a, a:0,0: a+b, a+b:0). Here the year five pupil could think and compare something out of the box. Surprisingly, he compared the eggs with different coloured eggs(green) which were not given in the task as shown in Diagram 5.

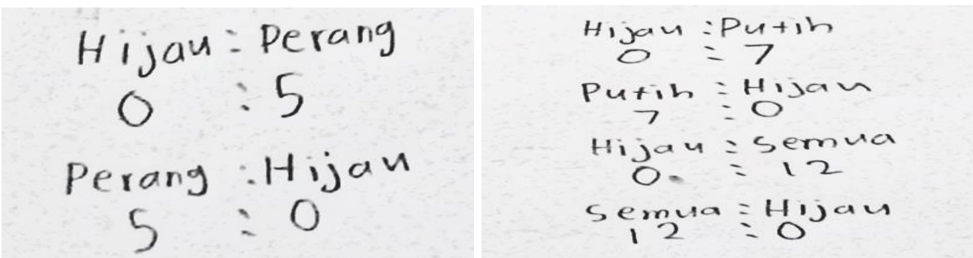

Diagram 5. Comparing with Non-Existing Objects

Conceptual Methods

A few respondents stated that a ratio can have another equivalent ratio. In Problem 1, two respondents stated that the ratio can have another equivalent ratio by doubling both parts and supported it by saying that the equivalent ratios are proportional. They stated that ratio four to five is equal to ratio eight to ten and they are equivalent ratios. In Problem 4, the respondents said that they could find other ratios and only need to multiply the numbers in the ratio with the same number for both the parts. The respondents formed new ratios by multiplication relationship (*ka*: *k*b) through doubling and multiplying with the same integer for both the parts. These operations were performed on the terms of the ordered pairs to form other ratios. They supported, by saying that the equivalent ratios are proportional as shown in Diagram 6.

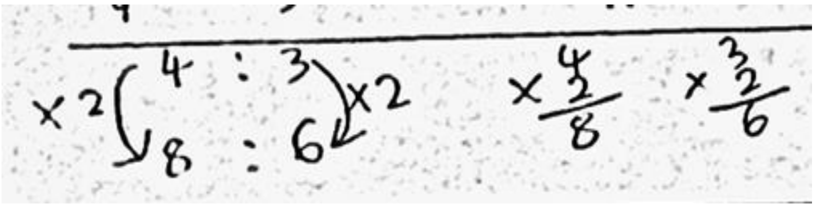

Diagram 6. Using Multiplication to Equal the Ratios

Further it was observed that Respondent 3 had the idea of between ratios. He stated ratio one to two first. Next, he multiplied both the parts of ratio with sixteen and performed another pair of ratios. Here, he used the between strategy among the terms or performed a scalar relationship. The algebraic expression can be defined as a: b = m (c:d). To form other ratios, he had used multiplication on the first and second term respectively of the ordered pair. He explained further the method to find other ratios. He only needs to multiply the numbers in the ratio with one same number for both parts. Furthermore, the scalar relationship was used in finding the missing value of between ratios in Problem 1, 3, 4 and 6.

… Times with two for many times… two cups' times two, four, four times two, eight, eight times two will be sixteen…the same goes for the other side… four times two, eight, eight times two sixteen, sixteen times two, thirty-two.

(R3,14.11.18, 11)

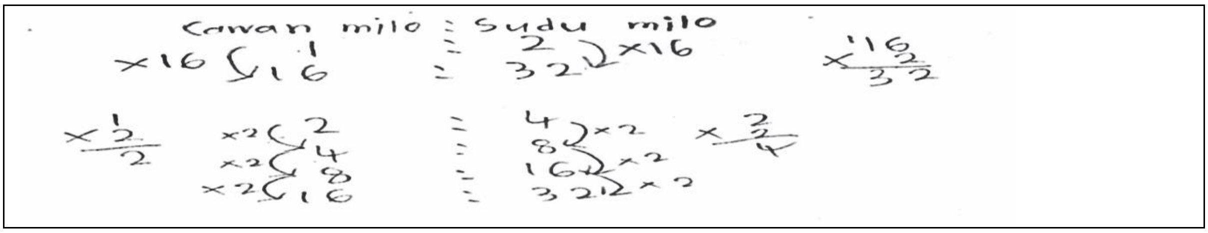

Diagram 7. The idea of between Ratios using Multiplication

In Problem 3 it was observed that the division operation was used to simplify the ratios as in Diagram 8.

…The number of pupils for walking is twenty-two and the number of pupils in the class is twenty-eight. Divide both sides with two. It will be ratio eleven to fourteen.

(R3,8.11.18,13)

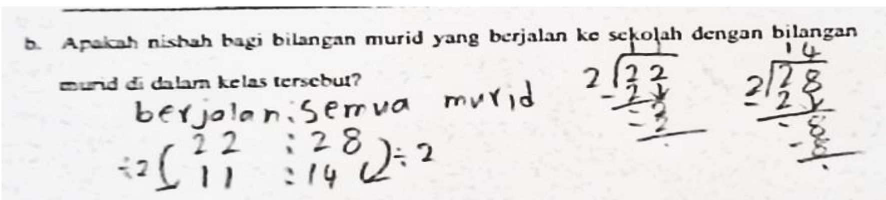

Diagram 8. The Idea of Simplifying Ratios Using Division

One respondent said that the numbers in the ratios can be *simplified* if they were in the same times table. A few respondents multiplied the ratios to form bigger ratios after simplifying the ratio at the beginning. For example, they simplified the ratio as c: d. Then it was defined, c: d = *m* (a: b) with *m*=3. After simplifying, the respondents multiplied the produced ratio with any number, simultaneously, to both the terms and produced ratio with bigger numbers. In this situation, division and multiplication were used simultaneously. Further, they used the multiplicative relationship to form ratios. The following diagram illustrate this point.

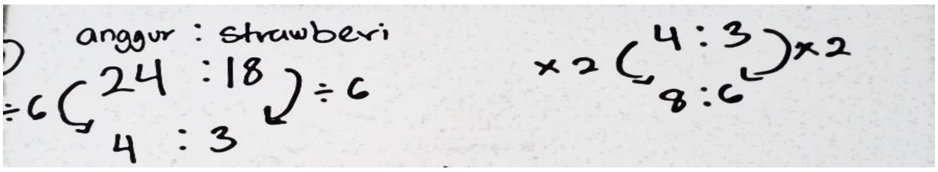

Diagram 9. Solving with Multiplication and Division to Form Equivalent Ratios

Similarly, another respondent connected the terms in ratio twenty-four to eighteen as numbers in the six times table and the numbers can be simplified to the numbers in the times table. She justified the reason why she had to *divide* by six. She stressed that the two numbers twenty-four and eighteen are numbers in the six times table.

Furthermore, ratios were written in the form of fractions to solve problems. Respondents used words like "one over five students did not take breakfast" instead of saying ratio one to four" which represents the part-to-part ratio. The fraction one over five gives meaning to part-to-whole ratios. Diagram 10 illustrates this point.

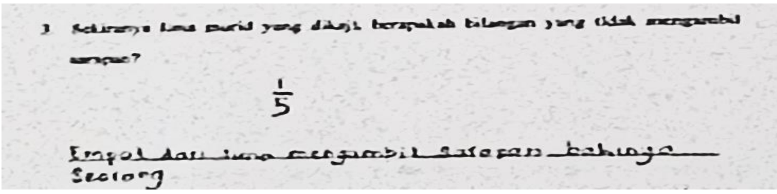

Diagram 10. Solving with Fraction

A few respondents used *equivalent fractions* to get the proportions. The respondents have multiplied the numerator and denominator four over five with the same factor to get eight over ten [k(a/b) = c/d]. They multiplied the numerator and denominator with an integer as shown in the diagram below.

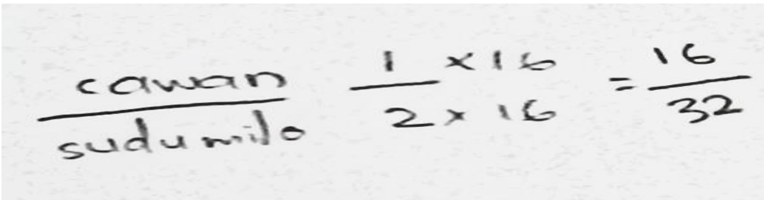

Diagram 11. Solution using Equivalent Fractions

Further, the respondents used the relationship in finding the missing value in Problem 1. The respondent had the idea of finding the *relationship of four numbers of quantities.* [ka: kb =c: d] They wrote two ratios with one missing number and solve the problem using the multiplication relationship as shown in Diagram 12.

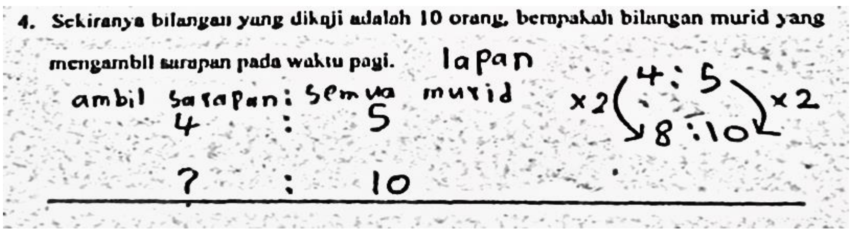

Diagram 12. Solving with Relationship of Four Numbers of Quantities

In Problem 3, the respondents had the idea of proportion whereby they had to equal two ratios and at the same time to find the fourth unknown value. ($\frac{a}{b} = \frac{c}{d}$). Algebraically, a and b are the first ordered pair and c and d of the second. For

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example, the ratio of students who ride the bus to the total number of students in the class will be ratio six to twenty-eight (a:b) and the respondents wrote in the form of a fraction $\frac{a}{b}$ which is $\frac{6}{28}$. The respondents need to solve the equivalent ratio $\frac{c}{d}$ as $\frac{c}{336}$. .The conservation of equivalence is preserved in both situations for the ratios as shown in Diagram 13.

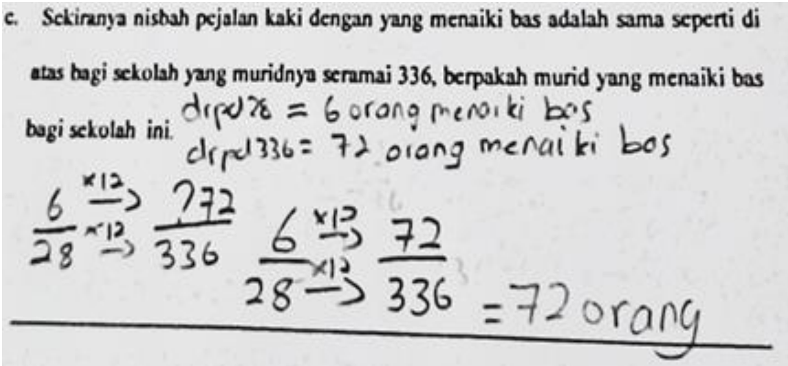

Diagram 13. Solving with Proportional Relationship

CONCLUSIONS

Both procedural and conceptual methods were identified in this study. Pupils had the idea of part-to-part and part-to-whole while writing down the ratios. These two subconstructs are important in acquiring the knowledge of ratio and proportion. Even though the procedural ideas of solving ratio and proportion word problems were transparent in this study, however, some translucent ideas using the conceptual methods were observed. The year five pupils solved the word problems using equivalent ratios and simplified ratios and were able to relate them to proportions. Missing value was figured out using the relationship of four numbers of quantities.

They were able to solve the basic problems of ratio and proportion using procedural and conceptual methods based on their own problem-solving skills (Debrenti, 2015). Some of the out of the box thinking emerged in this study. This consists of the stating ratio as comparison of three quantities which is called as extended ratio (Petit et al.,2020) and comparing a quantity with zero.

However, the higher academic level pupils were able to use the conceptual knowledge far better than the lower academic level pupils. They were able to make sense of ratio and relate ratios with proportions with reflective abstractions. The importance of prior knowledge of multiplication and division in building an initial understanding of proportional relationships were drawn from this study. Especially the findings can be a guideline to teachers out there who are seeking knowledge and methods in carrying out their teaching lessons. Teachers should have a clearer picture of how ratio and proportion should be taught, and pupils' ideas can be considered in acquiring the knowledge of ratio and proportion. The pupil's thoughts on ratio and proportion should be considered and should become part of the mathematics curriculum. Moreover, future research can be focused on different levels of students and standards such as the secondary or tertiary students. The methods used by them to solve ratio and proportion problems can be investigated using other research designs and methodologies.

The approach of learning that emphasizes memorization and rote learning should be substituted for a more reformed approach that includes dynamic discussion and effective classroom instruction that may lead to the construction of mathematical ideas by the learner. Therefore, we might want to consider whether the methods used by teachers in the present teaching and learning are sufficient for a child to learn ratio and proportion successfully. The teacher's role is now to create an environment where pupils are asked to give justifications and consider their informal ways of solving ratio and proportion problems (Lobato et al., 2014). Assorted types of problems can be posed, and we should give students room to determine solutions on their own. Since ratios and proportions are based on multiplicative relationship therefore the success of learning this concept is through multiplication and division (Petit et al., 2020). Sometimes a student might have their way of style in thinking to solve problems which the teacher could not think about it. Finally, children need to develop the multiplication concept and consider this very important as it is the part and parcel of learning ratio and proportion.

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